New Load Flow Method for Three Phase Radial Distribution Networks With Data Uncertainties

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Abstract: Load flow studies are done by electrical power distribution engineer to expand and monitor the Radial Distribution Network (RDN). This work presents an Affine Arithmetic (AA) with a flavour of Interval Arithmetic (IA) based load flow method to solve Data Uncertainty (DU) problems in micro as well as macro Distribution Networks (DNs) involving storage elements, sources connected to loads. It is considered that uncertainties of load flow data can be measured or estimated for balanced and unbalanced DNs. In order to solve with the iterative divergence issue of AA division operation during load flow solution, IA division convergence operation is effectively utilized to estimate the precise range-solution. The method can also handle simultaneous presence of DUs in load and line parameters. Simulation results obtained for 4 bus, 33-bus bus having balanced system and 8-bus, 11 having bus unbalanced system

Keywords: Radial Distribution Network, Affine Arithmetic, Interval Arithmetic, Data Uncertainties.

I. INTRODUCTION

Load flow study performed on a network is often uncertain due to errors in measurement. Some more study on range-load flow study is probably needed to devise a method which can provide an efficient load-flow interval or range-solution. A Distribution Network (DN) is subjected to Data Uncertainties (DUs) like, variation in system components such as generating capacities, lines, shunts, loads, tap-changing transformers etc. Hence, it is not economically feasible to employ actual-time monitoring and control devices everywhere along lateral, sub-lateral lines and loops of DNs.

The specified DUs of the DNs in load flow studies can be assigned into Interval numbers as per Interval Arithmetic (IA) to obtain variation in voltage and phase angle. L.V. Barbozaet et al [1]-[3] reported interval load-flow solution only for transmission network using IA tool [4]-[5]. The IA based load flow problems are validated and found computationally superior compared to traditional *Monte Carlo* simulations and *Stochastic* methods [6]. A self-validated IA tool has the strength to consider numerical round-offs. IA operations with probabilistic solution can be seen as a different case of *Fuzzy-set* operation with possibilistic solutions, which leads to more computations [7]. The probabilistic and possibilistic solutions are not easily comparable, because the reported assumptions on the data are different as one is qualitative; the other is quantitative, though random.

The Fuzzy set of [7] was applied by D. Das [8] to formulate IA based non-iterative algorithm, which is suitable for balanced radial network. For unbalanced RDNs, IA base interval solution was reported by B. Das [9] and Vaccaro et. al [10]. A small number, without a probability order represents each quantity in the numerical computation of IA. This leads to closed bound solution. The interval-solution that is obtained from IA computation for the computed function is of *wider range* than the *exact range* [11]. This infers that, to obtain an acceptable solution, IA exploits the use of data variation within relatively small-interval resulting in the use of an algorithm which decreases the time of computation but has lower accuracy.

Some problems of IA such as statistical dependency and accuracy tuning variables can be solved using Fuzzy Distribution Power Flow (FDPF) [12]. Here, simultaneous presence of DUs in: load model coefficients; network data; load forecast and bus shunts are considered. In FDPF, it is necessary to understand two algorithms to obtain the qualitative solution viz., Network Topology Power Flow (NTPF) [13] and Boundary Power Flow (BPF) [14]. In FDPF, input variables are modeled as Fuzzy/Interval numbers. To obtain accurate interval-solution, non-statistical limited small-range data is considered. Usually, solution set is non-singular in nature. Also, in case of FDPF, the choice of fuzzy membership function is often biased, as there are no well-known standards to select a membership function. Because of this, depending on the choice of membership functions (even within the same maximum and minimum), multipoint solutions exists. Self-validated Affine Arithmetic (AA) [11] tool keeps track of accuracy from the *centre* values of the interval numbers thus identifying the exact variation in the range for the computed quantities. AA uses first order approximations, with error generally being quadratic in width of the input intervals, for the quantity representations. In comparison to IA, the variables can be chosen easily to operate within accurate interval or range based on the center value, to yield lower width, resulting in higher operating cost for AA but with higher asymptotic accuracy.

The AA and IA based proposed method directly uses a non-interval load flow method [15]. This method is chosen because of its flexibility to add system components with DUs and also is computationally faster load flow method than NTPF [13]. Dharmasa et. al [18] proposed the method and implemented it for 15 and 27 bus data system, suggesting a possible extension for a three phase RDN.

Section 2, presents range relations between IA and AA; Section 3 gives an arrangement of line and load DU for DNs. Section 4 gives the formulations of Range-voltage equations for RDN. Sections 5 presents the results and discussion.

II. RANGE RELATION BETWEEN IA AND AA

If an interval number K_I is represented in IA [5] form as, $K_I = [K_l, K_u]$, then IA addition, subtraction, multiplication and division of $x_I = [x_l , x_u]$ and $y_I = [y_l , y_u]$ are :$x_I + y_I = [x_l + y_l , x_u+ y_u]$; $x_I – y_I = [x_l - y_u, x_u - y_l]$; $y_I = [min(x_l y_l, x_l y_u, x_u y_l, x_u y_u), max(x_l y_l, x_l y_u, x_u y_l, x_u y_u)]$; x_I ö $y_I = x_I$ò$(y_I)^{-1}$; *if 0*Č $[y_l,y_u]$. Some algebraic laws on real numbers with valid intervals in IA holds good viz., addition and multiplication are associative and commutative. However, distributive law, in some cases leads to over estimation [5].

A. Conversion from AA to IA:

Every affine form, which is a first-degree polynomial i.e. $\hat{x} = x_0 + x_1£_1 + \ldots + x_n£_n$ implies an interval bound for the corresponding ideal quantity x: namely, xČ$x_I = [x_0 – r$ _ $x_0 + r]$, where r is the total deviation of $\hat{x}, \sum_{i=1}^{n} |x_i|$. This is the smallest range that contains all possible values of AA of x, assuming that each $£_i$ ranges independently over the interval $U= [-1$ __ $+ 1]$. The ranges of x_I must be round outwards and this conversion discards all the correlation information present in the x of AA.

B. Conversion from IA to AA:

Every ordinary interval bound $x_I = [a$ _ $b]$ for an ideal quantity x can be represented in AA-form $\hat{x} = x_0 + x_k£_k$, where $x_0 = (a + b)/2$ is the centre point of x_I; half-width $x_k = (b - a)/2$, and $£_k$ is a 'new noise symbol, not occurring in any other existing AA-form. $£_k$ represents the uncertainty in the value of x that is implicit in its range x_I. Again, note that x_0 plus x_k are rounded carefully to obtain new AA-form, like the interval x_I, carries no correlation information.

C. Example: Multiplication Operation

If affine current is $\hat{B} = 30 - 4£_1 + 2£_2$ and resistance as $\hat{R} = 20 + 3£_1 + 1£_3$, then affine or range voltage $\hat{V}_{drop} = \hat{B} \times \hat{R}$ can be calculated as:

$$A(£_1 \ldots £_n) = 600 + 10£_1 + 40£_2 + 30£_3$$

$$Q(£_1 \ldots £_n) = (-4£_1 + 2£_2)(3£_1 + 1£_3)$$

Where A is first order affine product, Q is pure quadratic residue and $£_I$ is the shared noise symbol which partially correlates the affine operands. The two independent intervals of Q are: [-6_+6] and [-4_+4]. An estimate for the interval of Q results in [-24_+24], i.e. 0+24ε_4, in the first order affine format. The 'affine voltage drop' now is

$$\hat{V}_{drop} = 600 + 10£_1 + 40£_2 + 30£_3 + 24£_4$$

resulting in 600 ± 104 = [496_704]. However detailed AA analysis by Vaccaro et. al [10] shows that the actual range of Voltage drop $\hat{V}$drop is [528_675]; therefore, the range implied by AA is (704− 496)/(675 − 528) = 1.42 times wider than the exact range. It may be noted that the $\hat{V}_{drop}$ calculated with standard IA= [24_36]×[16_24] = [384_864], which is (864 − 384)/(675 − 528) = 3.26 times wider than the actual range. Negative correlation between $\hat{B}$ and $\hat{R}$ implied by the shared symbol $£_I$ leads to large discrepancy. The correlated terms −80$£_I$ and +90$£_I$ nearly cancel out in the AA calculation, but are added with the same sign in the IA. Also, AA traces most of the uncertainty in the original data by tuning the noise variables i.e. $£_1$, $£_2$ and $£_3$. Further, the loss of information or approximation error is introduced by the AA operation by considering residual term 24$£_4$.

In case of computation of IA-voltage drop, it is the product of interval current B_I and resistance R_I

$$V_{drop\,I} = B_I \times R_I =$$

$$[\min(B_lR_l, B_lR_u, B_uR_l, B_uR_u), \max(B_lR_l, B_lR_u, B_uR_l, B_uR_u)]$$

The source of its uncertainty in the IA voltage drop [384_864] = 624 ± 240 solution for the same operands is not specified. Hence, all of it must be regarded as approximation error of the operation.

B. IA verses AA Multiplication Operation View

IA and AA multiplication operation results are tabulated in Table 1. It can be observed that there is a better range estimation with AA, which is due to co-relation operands and quadratic residue, which are not considered in IA. Over-prediction/estimation factors occur in IA which is due to cascaded computations i.e., results of one-step affect inputs for the next step. The shared correlation operator $£_I$ in first order approximation form of AA and noise symbol $£_4$ of quadratic residue play vital role in obtaining range-solution near the actual values.

Table I: Comparison of range voltage drops

Content/ Tool	Detailed AA	IA	AA
Voltage drop	[528_675]	[384_864]	[496_704]
Relative over-estimation	1	3.26	1.42
Quadratic residue	$£_n$	Not considered	$£_4$
Noise Symbols	$£_1, £_2 \ldots £_n$	Not considered	$£_1$

Stofi et. al. [10] discussed that fewer function evaluations are needed in tighter range estimates of AA calculations which reduces the total running time.

III. DATA UNCERTAINITIES IN DISTRIBUTION NETWORKS

A line, with impedance Z^{abcg} of 3 phases (*abc*) DN and with DUs is represented using IA and AA as:

$$\left[Z^{abc}_{xr\,I}\right] = \begin{bmatrix} Z_{aaI} & Z_{abI} & Z_{acI} \\ Z_{baI} & Z_{bbI} & Z_{bcI} \\ Z_{caI} & Z_{cbI} & Z_{ccI} \end{bmatrix}$$; Suffix = $_I$ for IA,

$$\left[\hat{Z}_{xr}^{abc}\right]=\begin{bmatrix}\hat{Z}_{aa} & \hat{Z}_{ab} & \hat{Z}_{ac}\\ \hat{Z}_{ba} & \hat{Z}_{bb} & \hat{Z}_{bc}\\ \hat{Z}_{ca} & \hat{Z}_{cb} & \hat{Z}_{cc}\end{bmatrix};\quad \text{Prefix} = \hat{} \text{ for AA}$$

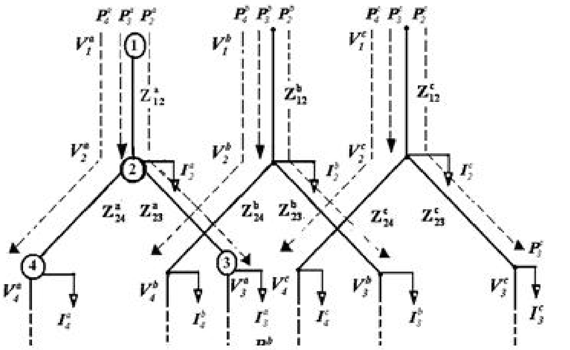

Fig. 1. *Load and line DUs of 3-phase typical RDN.*

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Fig. 1 shows the line and load Data Uncertainties (DUs). For simplicity, an assumption of balanced system is made and is extended to explain the unbalanced case.

A. Line Impedance equation in IA form:

IA line impedances with DUs from Fig. 1. can be arranged in diagonal matrix format as.

$$\left[Z_{IR}\right]=\left[Z_l,Z_h\right]=abs\begin{bmatrix}Z_{(j,i)I} & 0 & 0\\ 0 & Z_{(j,i+1)I} & 0\\ 0 & 0 & Z_{(j,i+n)I}\end{bmatrix} \quad (1)$$

Where, $i, i+1, i+2,\ldots,n$ = Incremental order of receiving end nodes

j = Sending end nodes

$[Z_l]$ = Lower range impedance of *abc* phases; $[Z_h]$= Higher range impedance of *abc* phases

B. Line Impedance equation in AA form:

Similarly, the AA form of line impedances with DUs can be expressed as (2) as first order AA-form of impedances.

$$\left.\begin{aligned}\hat{Z}_{ji} &= Z_0 + Z_{(ji)1}£_1 + \ldots + Z_{(ji)n}£_n\\ \hat{Z}_{i,i+1} &= Z_0 + Z_{(i,i+1)1}£_1 + \ldots + Z_{(i,i+1)n}£_n\\ \hat{Z}_{jn} &= Z_0 + Z_{(jn)1}£_1 + \ldots + Z_{(jn)n}£_n\end{aligned}\right\rangle \quad (2)$$

Where, Z_0 = Centre-point value; $£_1$, $£_2$….$£_n$ are co-relators

C. Load Current equation in IA form:

The absolute values of IA load currents are expressed as the ratio of conjugate of interval power to interval voltage.

$$\begin{bmatrix}I_{iI}^{abc}\\ I_{i+1I}^{abc}\\ I_{nI}^{abc}\end{bmatrix}=abs\begin{bmatrix}S_{iI}^{abc}/V_{iI}^{abc}\\ S_{i+1I}^{abc}/V_{i+1I}^{abc}\\ S_{nI}^{abc}/V_{nI}^{abc}\end{bmatrix}^* \quad (3a)$$

Where, V_{iI} = Interval voltage; S_{iI} = Interval power

Then, IA load injection with lower and upper bound is computed using division operation [5].

$$I_{iI} = abs\left[S_l \times (1/V_h), S_u \times (1/V_l)\right] \quad (3b)$$

D. Load current equation in AA form

In AA form, load current DUs is the ratio of conjugate of affine power to affine voltage. First order current and voltages expressed in AA form are:

$$\hat{I}_i^{abc} = abs(\hat{S}_i^{abc} / \hat{V}_i^{abc})^* \quad (4a)$$

$$\left.\begin{aligned}\hat{S}_i &= S_o + S_1£_1 + S_2£_2 + \ldots + S_n£_n\\ \hat{V}_i &= V_o + V_1£_1 + V_2£_2 + \ldots + V_n£_n\end{aligned}\right\rangle \quad (4b)$$

In (4b) S_0 and V_0 are centre-point values of power and voltage respectively and $£_1$, $£_2$…$£_n$ are co-relation symbols. According to the rule of unary operation the AA reciprocal form of operands as per Miyajima et al [17] is expressed as:

$$\begin{aligned}\frac{1}{\hat{V}_i} &= -\frac{1}{V_lV_h}V_o+\frac{V_l+V_h+2\sqrt{V_lV_h}}{2V_lV_h}+\left(-\frac{V_1}{V_lV_h}\right)£_1+\ldots\\ &+\left(-\frac{V_n}{V_lV_h}\right)£_n+\frac{V_l+V_h-2\sqrt{V_lV_h}}{2V_lV_h}£_{n+1}\quad (V>0)\end{aligned} \quad (4c)$$

It was observed that iterative division process using (4c) of AA, solution width diverges. This problem was overcome by using IA division of (3b) and re-converting the, IA load currents of (3b) back to first order AA.

$$\hat{I}_i = I_o + I_1£_1 + I_2£_2 + \ldots + I_n£_n \quad (4d)$$

In this way, adoption of AA would yield lower width when compared to IA.

IV. VOLTAGE RELATIONS FOR RDN WITH DUs

Expressing the voltage drops for the network in fig. 1. from source as product of down-stream branch currents *B* and impedances *Z*, then nodal voltage can be computed applying KVL as:

A. Node Voltage Relations in IA form

IA based voltage drop equation for balanced RDN is

$$[VD_{jiI}] = \sum_{i=2,3,..n}^{path\ nodes} [Z_{jiI}] \times \sum_{i=2,3...n}^{path\ nodes} [B_{jiI}]$$

(5)

In (5), as per concept of duality [15], separating Forward Path [*FP*] from $\sum_{i=2,3,..n}^{path\ nodes} [Z_{jiI}]$ and Backward Path [*BP*] from $\sum_{i=2,3,..n}^{path\ nodes} [B_{jiI}]$ it is found that *FP* and *BP* (*FP's* transpose) are dual. Then modified three–phase vector of IA voltage drop and nodal voltage suitable for unbalanced RDN are:

$$[VD_{ijI}^{abc}] = [FP][Z_{IR}^{abc}] \times [BP][I_{iI}^{abc}]$$

(6)

$$[V_{iI}^{abc}] = [V_{1}^{abc}{}_{I}] - [FP][Z_{IR}^{abc}] \times [FP]^T [I_{iI}^{abc}] \quad (7)$$

Where, $[ILF_R]^{3\phi}$= 3 - Phase *Interval Load Flow* matrix

In (7) [*FP*] act as network information matrix containing the values *1* and *0*, where *1* indicates presence of an element (eg. 2nd Path phase a) and *0* indicates absence of an element. To build matrix [*FP]* the procedure is to trace the path impedances Z^{abc}, i.e. follow nodal connectivity information of P_r^{abc} shown in Fig.1 as *r= 2, 3…i, i+1, i+2…n*.

$$[FP] = \begin{matrix} P_2^a \\ P_2^b \\ P_2^c \\ P_3^a \\ P_3^b \\ P_3^c \\ P_4^a \\ P_4^b \\ P_4^c \end{matrix} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (8)$$

And *FP* of (8), which is equal to the interval product of network impedances and branch currents.

$$[VD_l]^{3\phi}=min(min[(Z_{jil}B_{Il},Z_{jih}B_{Ih}),(Z_{jih}B_{Il},Z_{jih}B_{Ih}])^{\phi} \quad (9)$$

$$[VD_h]^{3\phi}=max(max[Z_{jil}B_{Il},Z_{jih}B_{Ih}),(Z_{jih}B_{Il},Z_{jih}B_{Ih}])^{3\phi} \quad (10)$$

$$[VD_I]^{3\phi}=[VD_l,VD_h]^{3\phi}$$

Then, IA nodal voltage of (7) becomes

$$\left[V_{iI}\right]^{3\phi} = \left[\left[V_{i1} - VD_{jih}\right]^{3\phi}, \left[V_{i1} - VD_{jil}\right]^{3\phi}\right] \quad (11)$$

Iterative IA mismatch voltage for RDN is

$$\left[\Delta V_{iI}{}^{k}\right]^{3\phi} = \left[\left[V_{il}{}^{k+1} - V_{ih}{}^{k}\right]^{3\phi}, \left[V_{ih}{}^{k+1} - V_{il}{}^{k}\right]^{3\phi}\right] \quad (12)$$

B. Nodal Voltage based on AA expression for RDN

AA based voltage drop equation for balanced RDN is

$$[\hat{V}_i] = [\hat{V}_1] - \sum_{i=2,3,..n}^{path\ nodes} [\hat{Z}_{ji}] \times \sum_{i=2,3...n}^{path\ nodes} [\hat{B}_{ji}] \quad (13)$$

In the same way, the 3ϕ AA voltage relation using [15]

$$[\hat{V}D_{ij}^{abc}] = [FP][\hat{Z}_{R}^{abc}] \times [BP][\hat{I}_{i}^{abc}] \quad (14)$$

The AA voltage of (14) contains line DU of (2a), load DU of (4a) and *FP* of (8).

$$[\hat{V}_i^{abc}] = [V_1^{abc}] - [FP][\hat{Z}_R^{abc}] \times [FP]^T [\hat{I}_i^{abc}] \quad (15a)$$

$$[\hat{V}_i^{abc}] = [V_1^{abc}] - [ALF]^{3\phi} [\hat{I}_i^{abc}] \quad (15b)$$

Where, $[ALF_R]^{3\phi}$= Three Phase Affine Load Flow Matrix

If the simplified (15a) affine nodal voltages at *i, i+1,…n* are expressed as the product of network impedances and branch currents, then, affine form of voltages are:

$$\begin{bmatrix} \hat{V}_i \\ \hat{V}_{i+1} \\ \hat{V}_n \end{bmatrix} = \begin{bmatrix} \hat{V}_j \\ \hat{V}_j \\ \hat{V}_j \end{bmatrix} - [FP] \begin{bmatrix} \hat{Z}_{ji} & 0 & 0 \\ 0 & \hat{Z}_{i,i+1} & 0 \\ 0 & 0 & \hat{Z}_{in} \end{bmatrix} [FP]^T \begin{bmatrix} \hat{I}_i \\ \hat{I}_{i+1} \\ \hat{I}_n \end{bmatrix} \quad (16a)$$

$$\hat{V}_i = \hat{V}_j - \hat{Z}_{ji} \hat{B}_{ji} \quad (16b)$$

$$\hat{V}_{i+1} = \hat{V}_j - \{\hat{Z}_{ji} \hat{B}_i + \hat{Z}_{i,i+1} \hat{B}_{i+1}\} \quad (16c)$$

General form of AA 3ϕ nodal voltage equation is

$$\begin{bmatrix} \hat{V}_i \\ \hat{V}_{i+1} \\ \hat{V}_n \end{bmatrix}_{(n-1)\times(n-1)}^{3\phi} = \begin{bmatrix} V_j \\ V_j \\ V_j \end{bmatrix}_{(n-1)\times 1}^{3\phi} - [A+Q]_{(n-1)\times 1}^{3\phi} \quad (16d)$$

Where, *A*= *1st* order AA value; Q = *2nd* order AA residue

The first order affine term *A* and quadratic residue term *Q* can be simplified using $£_{k=1...n}$ to compute the magnitude of affine nodal voltage as

$$A=[FP]\begin{bmatrix} Z_{ji0}\pounds_0 & 0 & 0 \\ 0 & Z_{(i,i+1)0}\pounds_0 & 0 \\ 0 & 0 & Z_{in0}\pounds_0 \end{bmatrix}[FP]^T\begin{bmatrix} I_{ik}\pounds_k \\ I_{(i+1)k}\pounds_k \\ I_{nk}\pounds_k \end{bmatrix}+$$

$$[FP]\begin{bmatrix} Z_{ji0}\pounds_0+Z_{jik}\pounds_k & 0 & 0 \\ 0 & Z_{(i,i+1)0}\pounds_0+Z_{(i,i+1)k}\pounds_k & 0 \\ 0 & 0 & Z_{in0}\pounds_0+Z_{ink}\pounds_k \end{bmatrix}$$

$$\times[FP]^T\begin{bmatrix} I_{i0}\pounds_0 \\ I_{(i+1)0}\pounds_0 \\ I_{n0}\pounds_0 \end{bmatrix};$$

$$Q=[FP]\begin{bmatrix} Z_{jik}\pounds_k & 0 & 0 \\ 0 & Z_{(i,i+1)k}\pounds_k & 0 \\ 0 & 0 & Z_{ink}\pounds_k \end{bmatrix}[FP]^T\begin{bmatrix} I_{ik}\pounds_k \\ I_{(i+1)k}\pounds_k \\ I_{nk}\pounds_k \end{bmatrix}$$

(16e)

Iterative 3ϕ*AA mismatch voltages* for RDN is

$$\left[\Delta\hat{V}_i^{\,k}\right]^{3\phi}=\left[\hat{V}_{il}^{\,k+1}-\hat{V}_i^{\,k}\right]^{3\phi}$$

(16f)

V. RESULTS AND DISUCSSIONS

The performance of proposed work, which is a blend of AA and IA tools, was tested on balanced 33-bus, 4-bus [16] and unbalanced 8-bus, 11-bus [13] DNs. In Fig. 2 from 8 and 11-bus system, both line and load data of DUs are considered simultaneously to obtain the interval solution.

A. Accuracy and computational speed

The qualitative results are comparable to established technique - like IA [6]. IA does not consider the co-relation operators, which leads to large errors discussed in Table 1. But IA is computationally faster compared to AA. Here it is found that the accuracy of AA tool increased compared to IA.

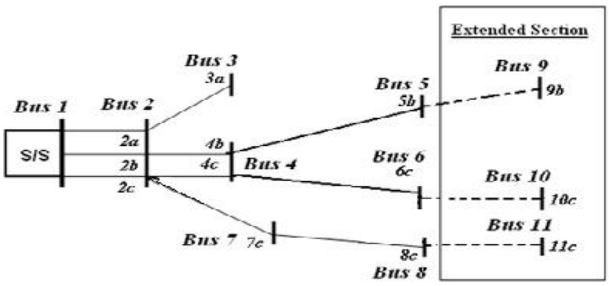

Fig. 2 *Unbalanced three phase RDN.*

The proposed method with high computational speed than IA will convergent fast than AA. In this work, iterative range-load currents of (4a) are computed using IA division (3b) instead of AA division. If iterative AA division operation [16] is performed using (4c), then it was found that the range widths are increasing by 0.02%, further results got diverged after 3rd iteration. For all the tests on the DNs with set accuracy of 0.001 the results in per unit are obtained by considering ±5% line DUs and ±10% load DUs, simultaneously.

B. Percentage Error Width and Crisp Difference

If in a loop with ±5% impedance, DU is introduced between 18th and 33rd node of balanced 33-bus DNs [15], then it was observed that improved voltage range-profile can be seen at nodes 10, 18, and 33. Table 2. shows AA range-solution for 33-bus, which is computed using (15a-16e). The table also compares converged narrow width AA solution with overestimated IA solution after 3rd iteration. To check the amount of decrease in the width of the voltage variation due to standard IA, a Percentage Width Reduction (PWR) term is defined i.e. (IA-AA)/IA×100. Crisp solution was calculated for IA and AA as per [12]. A minimum PWR of 25.86 occurred at 2A & maximum of 59.48 occurred at 6C for RDN. PWR is a measure of tuning of co-relation operators. In this work the co-relation operators of ± 0.2 to ± 0.9 are used for both the balanced and unbalanced DNs to obtain an efficient range-solution. The Crisp Difference (CD) between the two solutions (0.01 and 0.001 is set) is measured to validate the accuracy of solution.

C. Robustness and Flexibility with System Components: The robustness of AA iterative range-load flow program increased with the usage of IA division operation. The robustness test is conducted on balanced 33 bus, unbalanced 8 and 11 bus for DNs with DUs and results got converged at 3rd iteration.

Table II. Range Solution and Width Reduction for 33 Bus DNs

Bus No.	IA Solution		AA Solution		PWR %	CD
	V_l	V_h	V_l	V_h		
10	0.9104	0.9354	0.9233	0.9286	63.96	0.0031
18	0.8899	0.9207	0.9058	0.9123	78.89	0.0038
33	0.8930	0.9230	0.9084	0.9148	76.62	0.0036

Table III. Range Solution and PWR for 8 bus DNs

Converged Range-Solution for RDN at 3rd Iteration						
Ph-Bus	IA		AA		PWR %	CD
	V_l	V_h	V_l	V_h		
2 A	0.9811	0.9861	0.9827	0.9847	25.86	0.0001
2 B	0.9764	0.9827	0.9784	0.9809	32.76	0.0001
2 C	**0.9643**	**0.9739**	0.9673	0.9713	48.28	0.0002
3 A	0.9805	0.9856	0.9821	0.9842	25.86	0.0001
4 B	0.9726	0.9799	0.9749	0.9779	37.07	0.0001
4 C	0.9591	0.9702	0.9626	0.9671	56.90	0.0001
5 B	0.9716	0.9792	0.9740	0.9771	38.79	0.0001
6 C	**0.9573**	**0.9689**	0.9610	0.9657	59.48	0.0002
7 C	0.9631	0.9731	0.9662	0.9703	50.86	0.0002
8 C	0.9619	0.9722	0.9652	0.9694	52.59	0.0002

Observing the 6C bus in Table 3, it is found that IA range *changed* voltage at 6C during RDN i.e. from 0.9573 - 0.9739 to 0.9610 - 0.9657. Similarly, we can compare IA voltage change at bus 6C with AA of Table 3. In this work the AA co-relation operators of equation (2b) are varied ± 0.2 to ± 0.9, in equation (16) for RDN. These values are applied for both the balanced and unbalanced DNs to obtain an efficient range-solution.

VI. CONCLUSION

A new method based on Affine Arithmetic (AA) with the blend of Interval Arithmetic (IA) is able to obtain accurate interval solution for unbalanced with load and line DUs in the network. The method can evaluate uncertain data in the system by successfully formulating range-load flow equations for method is able to provide range-solution even with simultaneous operation of load and line data. The newly defined terms Percentage Width Reduction and Crisp Difference can be used as a measure of improvements in the load flow range-solution. The effectiveness of proposed method can be appreciated when we see more variation in PWR from source node to leaf end nodes. AA tool with flavor of IA has the potential to save time with better results in range-load flow, compared to using either AA or IA independently.

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